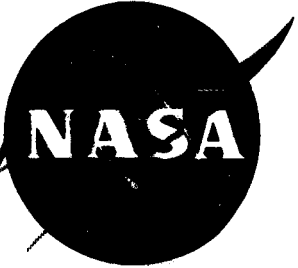


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TECHNICAL NOTE

D-801

A METHOD OF DETERMINING
AERODYNAMIC-INFLUENCE COEFFICIENTS
FROM WIND-TUNNEL DATA FOR WINGS AT
SUPERSONIC SPEEDS

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Langley Research Center
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SUMMARY

A method is described for determining aerodynamic-influence coefficients from wind-tunnel data for calculating the steady-state load distribution on a wing with arbitrary angle-of-attack distribution at supersonic speeds. The method combines linearized theory with empirical adjustments in order to give accurate results over a wide range of angles of attack. The experimental data required are pressure distributions measured on a flat wing of the desired planform at the desired Mach number and over the desired range of angles of attack.

The method has been tested by applying it to wind-tunnel data measured at Mach numbers of 1.61 and 2.01 on wings of the same planform but of different surface shapes. Influence coefficients adjusted to fit the flat wing gave good predictions of the spanwise and chordwise distributions of loadings measured on twisted and cambered wings.

INTRODUCTION

The influence-coefficient technique has been advanced as a means of predicting aerodynamic loads at supersonic speeds on wings having twist and camber or on wings experiencing aeroelastic deformations. This technique is usually based on the use of linearized theory for determining how local pressures will be affected by small changes in inclination of the wing surface at any point on the wing. Strictly theoretical methods, however, are handicapped by the failure of the theory to give reasonable pressure distributions, even for a flat wing, in many cases of practical interest. For example, one such case occurs when because of a rounded leading edge, a bow wave is formed ahead of a theoretically supersonic leading edge. Therefore, it is

often necessary to resort to semiempirical methods utilizing wind-tunnel pressure data.

One such method is described in reference 1. Although this method gave good results at small angles of attack, its accuracy was less at angles larger than $\pm 6^\circ$ because it did not attempt to account for the nonlinear variation with angle of attack in the available data.

The method of this paper employs a different concept in its approach to the determination of semiempirical influence coefficients and includes an approximate representation of the nonlinear effects exhibited by the experimental data. The end result is a single matrix equation which relates aerodynamic-load distribution to angle-of-attack distribution over a wide range of angles of attack with good accuracy.

The experimental data in reference 2 were used to demonstrate the method of this paper. Reference 2 presents the results of tests at Mach numbers of 1.61 and 2.01 of a series of five wing models of the same planform but differing in distribution of twist and camber. At the present time, reference 2 presents the only available data which are sufficiently detailed for evaluating such methods as that of reference 1 and of this paper.

The method of reference 3 was used for calculating the theoretical aerodynamic-influence coefficients, which are required as the basis for the empirical adjustments described in this paper.

SYMBOLS

$a(x,y), b(x,y)$	coefficients in equation for $\frac{4}{U} \phi(x,y)$ in terms of local angle of attack
c	local wing chord, in.
K, K', K''	correction factors
L	lift, lb
M	Mach number
m_o	tangent of leading-edge sweep angle
p_l	pressure on lower surface of wing, lb/sq in.
p_u	pressure on upper surface of wing, lb/sq in.

q	dynamic pressure, lb/sq in.
$[S\phi]$	matrix of influence coefficients for computing potential distribution from angle-of-attack distribution
$[S\phi]$	row matrix composed of elements taken from one row of $[S\phi]$
U	free-stream velocity of relative wind, in./sec
x, y	streamwise and spanwise coordinates, with origin of system at apex of wing, in.
x_{le}	local streamwise coordinate of wing leading edge, in.
x'	local streamwise coordinate measured from leading edge, in.
α_r	wing root angle of attack, radians
$\alpha(x, y)$	local angle of attack of mean camber line at point (x, y) , radians
$\beta = \sqrt{M^2 - 1}$	
$\frac{\Delta p}{q}$	lifting pressure coefficient, $\frac{p_l - p_u}{q}$
η	spanwise coordinate divided by semispan
Λ_{le}	wing leading-edge sweep angle, deg
$\phi(x, y)$	perturbation-velocity potential at point (x, y) , in. ² /sec
$ $	absolute value of enclosed quantity
$[]$	square matrix, number of rows equal to number of columns
$\begin{bmatrix} & \\ & \end{bmatrix}$	diagonal matrix, all elements zero except on principal diagonal
$[]$	row matrix
$\{ \}$	column matrix

Subscripts:

E	experimental
P	on one panel
T	theoretical

METHOD

In an analysis of the test data of reference 2, several observations were made which led to the method of this paper.

The chordwise pressure distributions for small angles of attack at $M = 1.61$, where the leading edge was subsonic, could be predicted with fair accuracy by linearized theory. However, the predicted pressures were too great in the vicinity of the leading edge, as is shown in figures 1(a) and 1(b). At higher angles of attack (figs. 1(c) and 1(d)) the differences between theory and experiment increased because of the nonlinear variation of the measured pressures with angle of attack. At $M = 2.01$, where the leading edge was supersonic, neither the magnitude nor the shape of the chordwise pressure distributions were very accurately predicted by linearized theory, as is shown in figure 2. The measured pressure distributions at $M = 2.01$ were similar in shape to those measured at $M = 1.61$.

The lack of correlation between theory and experiment at $M = 2.01$ would be expected from examination of the airfoil thickness distribution. All the wings tested for reference 2 had the NACA 65A005 thickness distribution. Although the 5-percent maximum thickness of this airfoil would ordinarily be considered within the scope of linearized theory, the rounded leading edge causes a detached shock wave, and shock waves are not considered in the linearized equations.

Qualitatively, the wing with detached shock wave has certain things in common with the wing with subsonic leading edge. The flow behind the shock wave is subsonic for some distance, and pressure disturbances on some portions of one surface of the wing can be transmitted to some portions of the other surface. The observed degree of similarity in the pressure distributions for the two cases might, therefore, be expected. Because a rigorous theoretical treatment of the wing with blunt supersonic leading edge is not available, the possibility of treating this case as a wing with subsonic leading edge was considered. For this purpose, a hypothetical wing with greater sweep angle, but otherwise the same in chord distribution as the actual wing, was substituted for the actual wing in the theoretical calculations. When the proper sweep angle was

used for this "equivalent" wing, the correlation between theory and experiment was very much improved. Although the agreement was not perfect, the remaining discrepancies were of a type which could be easily reduced by simple correction factors applied to theoretical influence coefficients calculated for the equivalent wing.

The method of this report, therefore, requires three steps. First, the equivalent wing sweep angle is found which gives theoretical pressure distributions of roughly the shape and magnitude measured on the flat wing. Second, this equivalent wing is then used in the calculation of influence coefficients by linearized theory. Third, these theoretical influence coefficients are further modified by empirical correction factors determined from the measurements made on the flat wing. These latter corrections include an approximate correction for nonlinearity in the variation of air load with angle of attack.

Determination of Equivalent Sweep

When the leading edge of the wing is subsonic, the equivalent wing is the same as the actual wing, and the necessary empirical corrections are applied to the theoretical influence coefficients for the actual wing. When the leading edge of the wing is supersonic, the equivalent wing-sweep angle is determined by the following procedures before the theoretical influence coefficients are calculated.

In order to determine the sweep angle required of the equivalent wing, use is made of the following equation derived from linearized theory:

$$\frac{\Delta p}{q} = \frac{16\alpha_r x}{\pi(m_0 + \beta)\sqrt{x^2 - m_0^2 y^2}} \quad (1)$$

Equation (1) was derived from equations (L1) and (L2) of reference 3. For that reason, it is only exact when $\beta/m_0 = 1.0$. By comparison with equation (7) of reference 4, it was found that the error in equation (1) is less than 5 percent when $\beta/m_0 = 0.4$. Equation (1) was used in place of the exact equation for the sake of consistency with the method of reference 3.

Equation (1) applies to flat triangular wings with subsonic leading and supersonic trailing edges. For wings other than triangular, it can be applied in regions not affected by the tip.

The object in applying equation (1) is to substitute experimental values of $\frac{\Delta p}{q}$, α_r , B , x , and y and to solve for the value of m_0

which satisfies the equations. The x coordinate of a given point must be expressed in terms of fraction of the chord $\frac{x'}{c}$ and m_0 in order to allow a point at a certain $\frac{x'}{c}$ and y on the actual wing to have the same location of $\frac{x'}{c}$ and y on the equivalent wing. Thus,

$$x = m_0 y + c \frac{x'}{c} \quad (2)$$

The solution of equation (1), with x expressed as in equation 2 may be found graphically by plotting $\frac{\Delta p}{q}$ against m_0 for constant $\frac{x'}{c}$, y , and α_r , taking as the solution the value of m_0 which gives the measured value of $\frac{\Delta p}{q}$ at that point on the wing. Several solutions may be obtained from points along the chord at the midsemispan, and an average of these values of m_0 may be used as the tangent of the equivalent wing sweep angle. Points on the leading edge must not be used because the theoretical value of $\frac{\Delta p}{q}$ at the leading edge is infinite.

The foregoing procedure was applied to the determination of a wing which would be equivalent to that tested in reference 2 at $M = 2.01$. The resulting leading-edge sweep angle was 63.4° instead of the actual value of 54° . Figure 2 shows the improvement which was obtained by using this value of sweep angle in the theoretical calculations. Figure 2(a) shows the chordwise pressure distributions at 50 percent semispan on the flat wing and on the wing with linear twist at a root angle of attack of 6° . In figure 2(b), the chordwise pressure distributions at 70 percent semispan are shown. Except at the leading edge, the shape of the theoretical pressure distribution on the equivalent wing ($\Lambda_{le} = 63.4$) is more nearly like the shape of the experimental distribution than is the shape of the theoretical pressure distribution for the actual wing. Furthermore, the ratio of theoretical pressure on the equivalent wing to measured pressure is about the same at a given point for both flat and twisted wings.

It is apparent, therefore, that the measured pressure at a given point on the twisted wing could have been closely approximated by first calculating the pressure on the equivalent wing with the same twist distribution and then multiplying that pressure by the ratio of measured pressure on the flat wing to theoretical pressure on the equivalent flat wing. (The fact that the theoretical pressure at the leading edge is infinite is of no concern when numerical methods are used, provided that points exactly on or too near the leading edge are not used.) The application of this observation to the adjustment of aerodynamic influence coefficients calculated by linearized theory for the equivalent wing is described in the sections which follow.

Theoretical Aerodynamic Influence Coefficients

A number of methods might be used for calculating influence coefficients for the equivalent wing by linearized theory, depending on the desired end result. In this report, the application of the results to aeroelastic calculations is considered foremost. For this purpose, aerodynamic influence coefficients are required in order to define the aerodynamic forces produced on small panels of the wing by deflection of a specific small panel. Reference 3 describes a method of calculating such coefficients which is economical of computing time, in that it does not require a direct calculation of pressure. The method of reference 3 was, therefore, adapted to the requirements of this report by certain modifications which are described in the appendix. These modifications were introduced to allow the final empirical adjustments to be made at the earliest convenient stage in the calculation of the force coefficients.

Some errors which were found in the equations of reference 3 are also noted in the appendix.

Empirical Adjustment of Theoretical Influence Coefficients

Inasmuch as reference 3 makes use of perturbation-velocity potential distributions for calculating aerodynamic influence coefficients, the following section will describe how the theoretical potential distributions, expressed as a function of angle-of-attack distribution, may be adjusted to agree with experimental data. The result will be a matrix equation relating the distribution of perturbation-velocity potential to the distribution of angle of attack. The method of adjustment will be described first for small angles of attack. The method will then be extended to cover a larger range by approximating the nonlinear variation of potential with angle of attack.

Small angles of attack.- According to linearized potential-flow theory, the perturbation velocity potential at a point $\phi(x,y)$ can be calculated from a measured distribution of lifting-pressure coefficient $\frac{\Delta p}{q}$ as follows:

$$\phi(x,y) = \frac{U}{4} \int_{x_{le}}^x \left(\frac{\Delta p}{q} \right)_y dx \quad (3)$$

The integration is from the leading edge to the point x , with y being held constant.

Equation 3 was used to calculate values of $\frac{4}{U}\phi(x,y)$ from the experimental data of reference 2 for comparison with theoretical values. The results showed that whether the wing was flat or twisted, theoretical values were in error by roughly the same percentage at small angles of attack. In other words, the actual potential at a point could be considered to be some constant times the theoretical potential, the constant being mainly dependent on the location of the point and very little dependent upon the source distribution which caused the potential. Thus,

$$\phi_E(x,y) = K\phi_T(x,y) \quad (4)$$

where the correction factor K can be derived from pressure measurements on a single wing model.

For convenience in correcting the potential distributions, a matrix of influence coefficients $[S\phi]$ was set up by the modified procedures of reference 3 described in the appendix to yield theoretical values of $\frac{4}{U}\phi_T(x,y)$ at the points required for calculating the panel loads. An element of this matrix is the value of $\frac{4}{U}\phi_T(x,y)$ at a point resulting from a unit angular deflection of one of the influence panels. Thus,

$$\left\{ \frac{4}{U}\phi_T(x,y) \right\} = [S\phi] \{ \alpha(x,y) \} \quad (5)$$

Equation (5) is adjusted to fit experimental data by multiplying the right side by a diagonal matrix $[K]$ whose elements are defined by equation (4). Thus,

$$\left\{ \frac{4}{U}\phi_E(x,y) \right\} = [K][S\phi] \{ \alpha(x,y) \} \quad (6)$$

Equation (6) is now easily solved to obtain the elements of $[K]$ which will give an exact fit to one set of experimental conditions of $\left\{ \frac{4}{U}\phi_E(x,y) \right\}$ and $\{ \alpha(x,y) \}$. Although there is no proof that equation (6) can then be applied to the calculation of distributions of potential corresponding to arbitrary angle-of-attack distributions, it has been found that equation (6) gives good approximations to the experimental data in reference 2 on twisted wings at small angles of attack, when $[K]$ is computed from flat-wing data.

Correcting for nonlinear variation with angle of attack.- At angles of attack above about 6° , the effects of nonlinearity began to be apparent

in the experimental data of reference 2. These effects varied from point to point on the wing. An approximate method for taking these effects into account follows.

There is no practical theoretical method of computing nonlinear aerodynamic influence coefficients corresponding to $[S\phi]$ in equation (6). It was necessary, therefore, to assume some arbitrary form for the relationship between potential and angle of attack, adjust it to fit the flat-wing data, and then test it against the available twisted and cambered wing data.

In order to aid in making a reasonable assumption as to the form of the influence-coefficient equation, flat-wing data of reference 2 were analyzed to determine what relationship existed between angle of attack and $\frac{4}{U}\phi(x,y)$ at each point. It was found that the potential at any point on the flat wing could be adequately expressed by a curve of the second degree in angle of attack with antisymmetry expressed in the following manner:

$$\frac{4}{U}\phi_E(x,y) = a(x,y)\alpha_r + b(x,y)|\alpha_r|\alpha_r \quad (7)$$

Since the second-degree term retains the sign of α_r , the curve of equation (7) is antisymmetrical. The coefficients $a(x,y)$ and $b(x,y)$ varied from point to point on the wing. The matrix formulation which expresses equation (7) for all points on the flat wing at which $\frac{4}{U}\phi(x,y)$ is required is as follows:

$$\left\{ \frac{4}{U}\phi_E(x,y) \right\} = [a(x,y)] \{1\} \alpha_r + [b(x,y)] \{1\} |\alpha_r| \alpha_r \quad (8)$$

Equation (8) expresses an empirical relationship for the flat wing only. The final form of the influence-coefficient equation must give the same results as equation (8) for the flat wing. Of several forms tested which met this requirement, the equation which worked best for twisted and cambered wings was the following:

$$\left\{ \frac{4}{U}\phi(x,y) \right\} = [K'] [S\phi] \{ \alpha(x,y) \} + [K''] [S\phi] [|\alpha(x,y)|] \{ \alpha(x,y) \} \quad (9)$$

The elements of the diagonal matrices $[K']$ and $[K'']$ are calculated by setting equation (9) identical to equation (8) when

$\{a(x,y)\} = \{1\}a_r$, in which case $\left[|a(x,y)|\right] = [1]|a_r|$. In order to satisfy the identity, coefficients of like powers of a_r must be equal. Thus,

$$[K'] [S\phi] \{1\} a_r = [a(x,y)] \{1\} a_r \quad (10)$$

and

$$[K''] [S\phi] \{1\} |a_r| a_r = [b(x,y)] \{1\} |a_r| a_r \quad (11)$$

From equation (10),

$$[K']^{-1} \{1\} = [a(x,y)]^{-1} [S\phi] \{1\} \quad (12)$$

or

$$\left\{\frac{1}{K'}\right\} = \left[\frac{1}{a(x,y)}\right] \left\{\sum [S\phi]\right\} \quad (13)$$

where $\left\{\sum [S\phi]\right\}$ is a column matrix, each element of which is the sum of the corresponding row of $[S\phi]$. From equation (11), in like manner,

$$\left\{\frac{1}{K''}\right\} = \left[\frac{1}{b(x,y)}\right] \left\{\sum [S\phi]\right\} \quad (14)$$

The elements of $[K']$ and $[K'']$ evaluated from equations (13) and (14) are used in equation (9) to calculate adjusted perturbation-velocity potentials resulting from any angle-of-attack distribution. A procedure for calculating lift distributions from equation (9) is given in the appendix.

Evaluation of Method

Equation (9) expresses an empirical relationship which is necessarily accurate for the flat wing. In order to demonstrate its usefulness for predicting the loads on twisted and cambered wings, influence coefficients were computed for 25 points on the wing planform tested in reference 2. The actual planform is shown in figure 3(a) and the equivalent wing at $M = 2.01$ is shown in figure 3(b). Each influence panel occupies 20 percent of the semispan and 20 percent of the chord.

At $M = 1.61$, the actual wing geometry (fig. 3(a)) was used in the necessary theoretical calculations. The elements of the theoretical matrix $[S\phi]$ for $M = 1.61$ are listed in table I. The matrix products of the numbers in a given row times the angles of attack at the centers of the panels indicated by the column headings will be the value of $\frac{4}{U}\phi_T(x,y)$ at the trailing edge of the panel designated by the row number.

The corresponding matrix $[S\phi]$ for the equivalent wing at $M = 2.01$ is in table II.

The data reduction procedure used to obtain the coefficients $a(x,y)$ and $b(x,y)$ in equation (8) may be outlined as follows: Chordwise distributions of $\frac{\Delta p}{q}$ on the flat wing for each angle of attack were integrated from the wing leading edge to the trailing edge of each panel by numerical integration procedures. Where necessary, interpolation was performed to obtain the integrals at the span stations corresponding to the centers of the influence panels. The pressure integrals were plotted against angle of attack and were fitted with second degree curves by least-squares procedures. (The foregoing procedures may be varied according to individual preference.) The coefficients $a(x,y)$ and $b(x,y)$ obtained in this manner, together with the K' and K'' coefficients derived from them by equations (13) and (14), are tabulated in table III.

Tables I, II, and III apply, of course, only to the wing tested in reference 2. They are included to aid in visualizing the results of the method of this report and the relative magnitudes of the quantities involved.

Values of $\frac{4}{U}\phi(x,y)$ were calculated by equation (9) for wings 1 and C of reference 2 for several angles of attack. The results of the calculations at constant fractions of the chord were plotted against span station and are presented in figures 4 to 7. (At $\frac{x'}{c} = 1.0$, $\frac{4}{U}\phi(x,y)$ is equal to the product of section lift coefficient times local chord.)

Figures 4(a) to 4(c) show spanwise distributions of $\frac{4}{U}\phi(x,y)$ for wing 1 at a Mach number of 1.61. Wing 1 had a built-in linear twist distribution. The angle of attack at any station η is given in radians by the equation

$$\alpha(x,y) = \alpha_r - 0.1047|\eta|$$

In addition to the values calculated by equation (13), experimental values of $\frac{4}{U}\phi(x,y)$ and those predicted by linearized theory are also shown.

Figures 5(a) to 5(c) are corresponding plots for wing C, which was not twisted but cambered and had an NACA $a = 0$ mean camber line, 4 percent high at each spanwise station.

Figures 6 and 7 show the spanwise distributions for wings 1 and C, respectively, at a Mach number of 2.01.

Since theoretical influence coefficients were not calculated for the actual wing at $M = 2.01$, the values of $\frac{4}{U}\phi_T(x,y)$ predicted by linearized theory are not shown in figures 6 and 7. The diamond-shaped symbols in these figures represent instead the values which would be calculated by the method of this report if the corrections for non-linearity were omitted. These points were calculated by the equation:

$$\left\{ \frac{4}{U}\phi(x,y) \right\} = [K'] [S\phi] \{ \alpha(x,y) \}$$

with the use of values of $[K']$ in table III for $M = 2.01$.

DISCUSSION

The method of this paper is not intended as a theory for explaining the behavior of wings with blunt leading edges at supersonic speeds. It is truly a curve-fitting procedure based on the following observations: First, that the equations of linearized theory can be made to give a better approximation to measured pressures by adjusting the leading-edge sweep parameter which appears in the equations; and second, that the approximation can be further improved by adjusting the influence coefficients calculated from the altered equations so that they conform to measurements made on a flat wing.

The degree of accuracy in predicting twisted and cambered wing loadings which may be obtained by basing the corrections on data from tests of a flat wing alone is illustrated in figures 4 to 7. In using these results to evaluate the application of the method to aeroelastic analysis, the accuracy with which aeroelastic deformations are represented by the twisted and cambered wing surface shapes must be taken into account. The accuracy with which these surfaces are approximated by the influence panels used must also be considered.

The greatest discrepancies at positive angles of attack were obtained in predicting the cambered wing loadings, especially at $M = 2.01$. A cross plot against x'/c of experimental and calculated points at positive angles of attack in figure 7 at, for example, 50 percent of the semispan ($\eta = 0.50$) would show a nearly constant error at each point along the chord. Examination of the influence coefficients showed that such an error could result from using angles of attack for the leading-edge panels which were not representative of the effective angles over the same portions of the cambered wing. Since there is a difference in angle of attack on the cambered wing of about 13.8° between the points $x'/c = 0.025$ and $x'/c = 0.20$, some error of this type might be expected.

Errors of the type just discussed could probably be greatly reduced by replacing each leading-edge influence panel by a number of smaller panels which would more closely approximate the large curvature of the cambered wing. Such large curvature would not be expected to result from camber bending due to airload. In view of this fact, and because of the great increase in the computations which would be required, no further analysis of the effects of adding more influence panels was attempted.

A number of modifications in the procedures described in this report are possible. For example, it may be desired to calculate pressure distributions rather than potential or load distributions. In that case, a theoretical matrix of influence coefficients for calculating pressure may be calculated by replacing the equations of reference 3 with corresponding equations for pressure. Correction factors would then be calculated in the manner described, substituting pressure wherever $\frac{4\phi}{U}(x,y)$ appears. Points exactly on the leading edges should not be used, as the theoretical pressure at these points is infinite. Although the data reduction would be simplified by using pressure coefficients, the theoretical computations would be more complicated than those required for potential coefficients.

The method of this report should give accurate predictions of the effects of deflecting leading- or trailing-edge control surfaces if the boundaries of one or more of the influence panels are made to coincide with the actual control-surface boundaries. Although only the application to spanwise symmetrical loads could be shown from the available data, it may reasonably be assumed that the correction factors obtained from symmetrical tests can also be applied to the theoretical influence coefficients for antisymmetrical airloads.

The possibility of applying some or all of the concepts of this report to the case of dynamic loading has not been investigated.

It would be desirable for any empirical method such as the one described to be further verified by tests on other wings and at other Mach numbers. Unfortunately, such tests are not available at the present time.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., February 6, 1961.

APPENDIX

THEORETICAL AERODYNAMIC INFLUENCE COEFFICIENTS

Reference 3 presents equations by which the elements of the matrix $[S\phi]$, defined by equation (5), may be calculated. The specific use of those equations for computing $[S\phi]$ is not described in reference 3 and will, therefore, be described in this appendix. Some corrections to the equations of reference 3 will also be noted.

Calculation of $[S\phi]$

A typical element of the matrix $[S\phi]$ is the theoretical change in $\frac{4}{U}\phi_T(x,y)$ produced at the center of the trailing edge of one of the influence panels by a unit change in deflection of another panel. The deflection of a panel can be simulated by adding and subtracting in a certain manner the deflections of four "control surfaces," as is shown in reference 3; therefore, the potential at a given point due to the deflection of the panel can be calculated by adding and subtracting in like manner the potentials at the point due to the deflections of the same control surfaces.

Each equation in the appendix of reference 3 defines the quantity $\frac{1}{q} \frac{dL_P}{dy}$ as the evaluation of a certain function of x and y between two limits in x , which are the leading and trailing limits of the panel on which the load is to be calculated. Since

$$\begin{aligned} \frac{1}{q} \frac{dL_P}{dy} &= \frac{4}{U} [\phi_T(x_2, y) - \phi_T(x_1, y)] \\ &= \frac{4}{U} \left[\phi_T(x, y) \right]_{x_1}^{x_2} \end{aligned} \quad (A1)$$

it follows that the equations for $\frac{1}{q} \frac{dL_P}{dy}$ may be regarded as equations for $\frac{4}{U}\phi_T(x,y)$ if the indicated evaluation between limits in x is omitted.

Corrections to Equations

Equations for perturbation velocity potential in supersonic flow must yield a value of zero along the leading edge of a surface and along certain other boundaries, such as the Mach lines extending from the vertex of a control surface with subsonic hinge line. It will be found that in many cases the equations of reference 3 do not satisfy these boundary conditions. In each equation where this error was found, it was traced to improper placement of an absolute value sign in the inverse hyperbolic cosine term (or terms). In each case, the absolute value sign should have enclosed the entire argument. For example, in equation (L1) of reference 3, the term which appears as

$$\cosh^{-1} \left[1 + 2 \frac{(x - m_0 y)}{(m_0 + \beta) |y|} \right]$$

should have been

$$\cosh^{-1} \left| 1 + 2 \frac{x - m_0 y}{(m_0 + \beta) y} \right|$$

Errors of the same type were found in equations (S1), both terms of (S2), (S3), (S4), both terms of (R2), and (L2).

Influence Coefficients for Aerodynamic Load

For the purposes of aeroelastic analyses, influence coefficients which relate lift force to angle of attack are usually required. Influence coefficients in this form are readily derived from equation (9).

The quantity $\frac{4}{U} \phi(x, y)$ is calculated by equation (9) for points at the center of the trailing edge of each panel. The difference between the values calculated by equation (9) for two adjacent panels at the same span station is the change in $\frac{4}{U} \phi(x, y)$ over the nearest of the two panels, which is also the quantity $\frac{1}{q} \frac{dL_p}{dy}$ at the center of that panel (eq. (A1)). A matrix $[D]$ may be formulated which, when pre-multiplied into equation (9), will give:

$$\begin{aligned}
\left\{ \frac{1}{q} \frac{dL_P}{dy} \right\} &= [D] \left\{ \frac{1}{U} \phi(x,y) \right\} \\
&= [D][K'] [S\phi] \{ \alpha(x,y) \} + [D][K''] [S\phi] \left[|\alpha(x,y)| \right] \{ \alpha(x,y) \} \quad (A2)
\end{aligned}$$

For the arrangement of rows used in tables I and II, the matrix, $[D]$ will be that given in table IV.

The panel lift per unit dynamic pressure, $\frac{L_P}{q}$, is calculated by a numerical integration of $\frac{1}{q} \frac{dL_P}{dy}$ between the spanwise limits of each panel. This integration is accomplished by premultiplying equation (A2) by an integrating matrix denoted by $[I_{int}]$. Thus,

$$\begin{aligned}
\left\{ \frac{L_P}{q} \right\} &= [I_{int}] \left\{ \frac{1}{q} \frac{dL_P}{dy} \right\} \\
&= [I_{int}] [D][K'] [S\phi] \{ \alpha(x,y) \} + [I_{int}] [D][K''] [S\phi] \left[|\alpha(x,y)| \right] \{ \alpha(x,y) \} \quad (A3)
\end{aligned}$$

The form of the integrating matrix will depend on whether the trapezoidal rule, Simpson's rule, or one of several other methods of numerical integration is preferred. Therefore no specific form is given for $[I_{int}]$.

REFERENCES

1. Zisfein, Melvin B., Donato, Vincent W., and Farrell, Robert F.: Supersonic Aeroelastic Effects on Static Stability and Control. Pt. I - Aerodynamics, Vols I and II. WADC Tech. Rep. 58-95 (ASTIA Doc. No AD 155739), U.S. Air Force, Dec. 1958.
2. Grant, Frederick C.: A Tabulation of Wind-Tunnel Pressure Data at Mach Numbers of 1.61 and 2.01 for Five Swept Wings of the Same Plan Form but Different Surface Shapes. NACA RM L58D23, 1958.
3. Woodward, F. A.: Aeroelastic Problems of Low Aspect Ratio Wings. Part II. Aerodynamic Forces on an Elastic Wing in Supersonic Flow. Aircraft Engineering, vol. XXVIII, no. 325, Mar. 1956, pp. 77-81.
4. Grant, Frederick C.: The Use of Pure Twist for Drag Reduction on Arrow Wings With Subsonic Leading Edges. NACA TN 4104, 1957.

TABLE I.- MATRIX OF THEORETICAL INFLUENCE COEFFICIENTS $[S_{ij}]$, DEFINED BY EQUATION (5), FOR WING OF FIGURE 3(a) AT $M = 1.61$

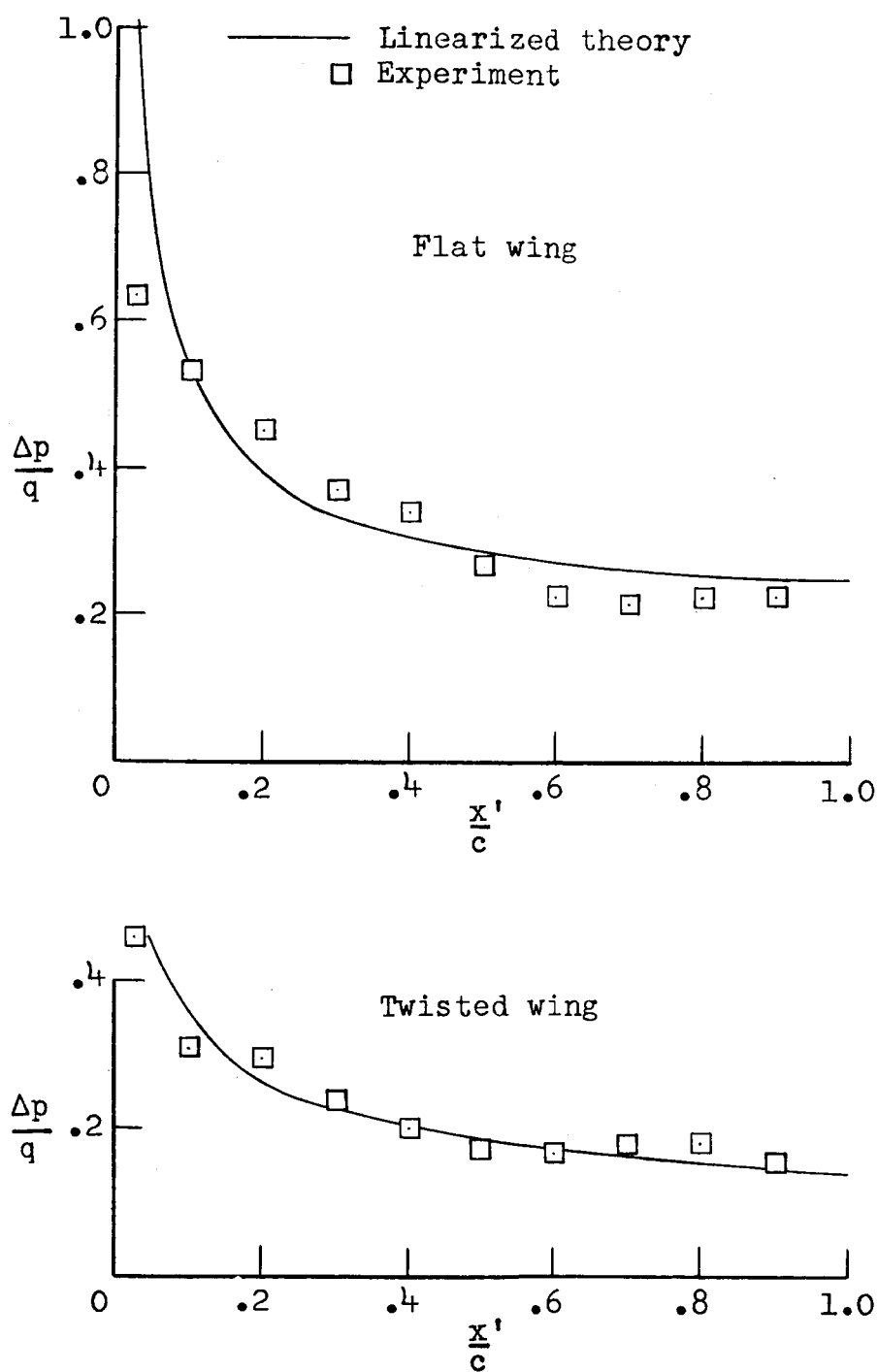
Panel no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	8.5435	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	5.0813	.4601	0	0	0	8.6968	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	3.5694	1.5001	0	0	0	5.3676	.6252	0	0	0	8.6659	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	2.2478	2.4690	0	0	0	3.8879	1.6917	0	0	0	5.4048	.8062	0	0	0	8.6406	0	0	0	0	0	0	0	0	0
21	1.5882	2.5658	.2711	0	0	2.4233	2.8623	0	0	0	4.0208	1.7608	0	0	0	5.4341	1.0165	0	0	0	8.6221	0	0	0	0
2	4.9528	6.4464	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	2.8082	3.0514	.0785	0	0	4.7207	6.6214	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	2.4957	1.5705	1.0156	0	0	3.3081	3.2515	.1687	0	0	4.0386	6.6455	0	0	0	0	0	0	0	0	0	0	0	0	0
17	2.1578	1.1768	1.2658	0	0	2.8900	1.7724	1.1502	0	0	3.4943	3.2619	.2839	0	0	3.2950	6.6763	0	0	0	0	0	0	0	0
22	1.4482	1.4808	1.3103	0	0	2.6976	1.2545	1.3948	0	0	2.9445	1.8088	1.2616	0	0	3.7945	3.2566	.4272	0	0	2.4066	6.7154	0	0	0
3	3.1942	3.7203	5.6381	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	2.1032	1.4019	2.6533	0	0	2.7883	3.6921	5.7805	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1.9447	.9374	2.1583	.4032	0	2.6113	1.9566	2.9230	0	0	1.8771	3.1707	5.7684	0	0	0	0	0	0	0	0	0	0	0	0
18	1.8783	.6993	.4714	1.4060	0	2.4748	1.3174	1.8564	.5600	0	3.4496	2.3317	3.0376	0	0	.1194	2.4795	5.7588	0	0	0	0	0	0	0
23	2.7710	.6104	.7358	.9569	0	2.1868	1.1960	1.1997	.9440	0	1.5858	1.4554	1.7613	.7336	0	3.6092	3.0085	3.1649	0	0	1.4497	5.7519	0	0	0
4	2.2708	2.2484	3.2162	4.7494	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1.6864	1.0379	1.2790	2.1604	0	1.7416	2.1021	3.1565	4.8560	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1.5664	.7226	.8682	1.7226	0	2.8916	2.3949	1.9494	2.4194	0	0	.6829	2.4933	4.7993	0	0	0	0	0	0	0	0	0	0	0
19	1.5355	.5599	.6558	1.1134	.3456	2.3228	1.3627	1.3039	1.9506	.0030	1.5839	2.4164	2.6569	2.5247	0	0	0	0	0	0	0	0	0	0	0
24	1.5239	.4550	.5207	.7616	.5442	2.1268	1.0555	1.0133	1.2671	.4432	2.7542	1.9383	1.4712	1.5673	.0536	0	1.3523	2.9743	2.6502	0	0	0	0	0	0
5	1.9872	1.4691	1.8020	2.5812	3.7241	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1.4300	.8967	.9422	1.1231	1.6364	.9159	1.1695	1.5954	2.4794	3.7886	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1.2771	.6212	.6581	.7753	1.2706	1.7187	1.8958	2.4132	2.1583	1.8927	0	0	0	1.4093	3.6660	0	0	0	0	0	0	0	0	0	0
20	1.2456	.4891	.5153	.5953	1.0708	2.3537	2.2237	1.5868	1.3276	1.4835	0	.1784	1.4172	2.6395	2.0152	0	0	0	0	0	0	0	0	0	0
25	1.2281	.3897	.4048	.4549	.8731	2.3365	1.3961	1.1644	1.0311	1.2736	.6005	1.4143	2.2894	1.6808	1.5448	0	0	.0282	2.2294	2.1823	0	0	.0276	3.3531	0

TABLE II. - MATRIX OF THEORETICAL INFLUENCE COEFFICIENTS $[S_1]$, DEFINED BY EQUATION (5), FOR WING OF FIGURE 3(b) AT $M = 2.01$

Panel no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	6.1004	0	0	0	0	0.7026	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	3.9734	0	0	0	0	7.0157	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	2.9699	.6184	0	0	0	4.4106	0	0	0	0	6.9488	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	2.3069	1.0212	0	0	0	3.3413	.7531	0	0	0	4.5120	.0070	0	0	0	6.8816	0	0	0	0	0	0	0	0	0
21	1.6032	1.4996	0	0	0	2.7662	1.1338	0	0	0	3.3564	.8502	0	0	0	4.5904	.0436	0	0	0	6.8145	0	0	0	0
2	2.7424	4.6038	0	0	0	1.4184	.6893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1.8049	2.3607	0	0	0	4.3867	5.5188	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1.5840	1.7509	.1637	0	0	2.4887	2.6831	0	0	0	3.8805	5.5005	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1.4641	.9651	.6973	0	0	2.1902	1.9988	.2450	0	0	2.6096	2.7574	0	0	0	3.3377	5.4824	0	0	0	0	0	0	0	0
22	1.3144	.7085	.8333	0	0	2.0529	1.2508	.7835	0	0	2.2544	1.9635	.3332	0	0	2.7688	2.8374	0	0	0	2.7391	5.4647	0	0	0
3	.6865	1.7780	3.9598	0	0	2.0414	1.3786	.6893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1.0566	.8354	1.9981	0	0	3.0222	3.5213	4.8591	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1.0106	.5476	1.5968	0	0	2.0045	1.5373	2.3086	0	0	2.2442	3.1354	4.8189	0	0	0	0	0	0	0	0	0	0	0	0
18	.9906	.3894	1.1106	.2615	0	1.7969	1.1042	1.9026	0	0	2.3204	1.7408	2.3789	0	0	1.2627	2.6739	4.7768	0	0	0	0	0	0	0
23	.9746	.2859	.6773	.5352	0	1.7126	.8795	1.3326	.3589	0	1.8889	1.1874	1.9493	0	0	2.8157	2.0760	2.4555	0	0	.0556	2.0561	4.7324	0	0
4	.5130	.5164	1.3434	3.2433	0	1.4713	1.3786	1.3786	.6893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	.5441	.2717	.6606	1.6140	0	2.3893	2.3841	3.1178	4.1306	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	.6050	.1714	.4358	1.2668	0	1.7877	1.3261	1.4516	1.9196	0	1.3533	1.7269	2.6613	4.0636	0	0	0	0	0	0	0	0	0	0	0
19	.6701	.1106	.3092	1.0739	0	1.5401	.9899	1.0577	1.5940	0	2.2897	2.1327	1.7297	1.9890	0	0	.4390	2.0726	3.9890	0	0	0	0	0	0
24	.7345	.0693	.2250	.8323	.1066	1.4167	.8138	.8520	1.3645	0	1.8474	1.2053	1.1650	1.5958	0	1.2247	2.0063	2.3410	2.0669	0	0	0	1.1106	3.9804	0
5	.3048	-.0007	.1400	.8086	2.4088	1.1159	1.2362	1.3767	1.3786	.6893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	.4297	-.0001	.0662	.4163	1.1887	1.6386	1.7405	2.0722	2.6385	3.2885	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	.5242	-.0005	.0275	.2716	.9121	1.44956	1.2569	1.3098	1.3641	1.5054	.5852	.8670	1.2595	2.0618	3.1792	0	0	0	0	0	0	0	0	0	0
20	.6044	-.5678	.5777	.1869	.7589	1.1664	1.4682	.4002	1.0100	1.1922	1.3196	1.5422	2.0038	1.9444	1.5822	0	0	0	1.0808	3.0559	0	0	0	0	0
25	.9682	-.0007	.0025	.1288	.6531	.7414	.7272	.5016	.8255	1.0297	1.8371	1.9942	1.4311	1.1770	1.2324	0	.0457	1.0954	2.2565	1.6779	0	0	0	.2827	2.9112

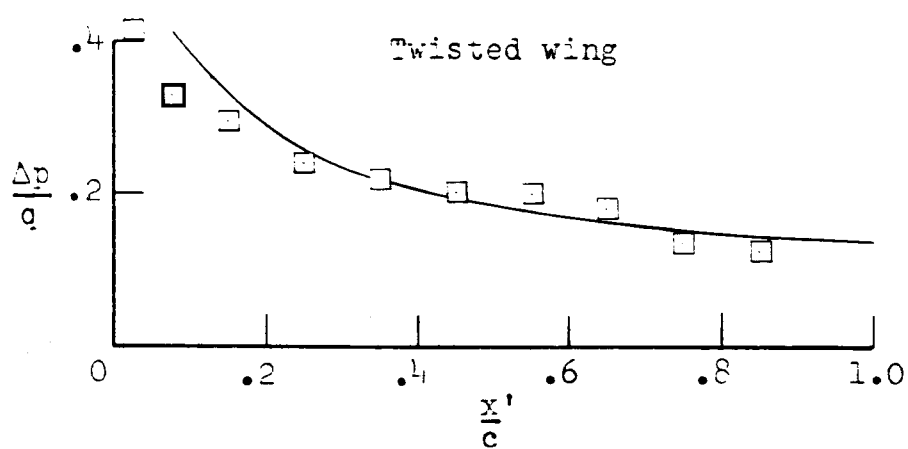
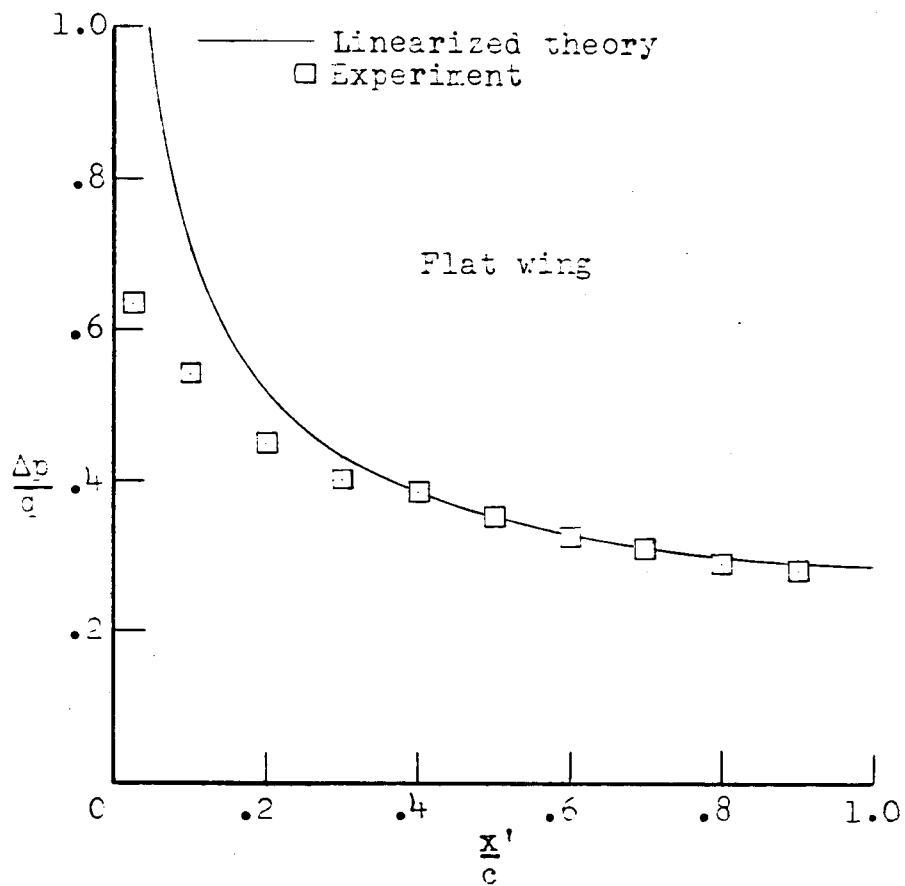
TABLE III.- ELEMENTS OF $[a(x,y)]$ AND $[b(x,y)]$ IN EQUATION (8) FOR DATA OF REFERENCE 2
AND ELEMENTS OF $[K']$ AND $[K'']$ FROM EQUATIONS (13) AND (14)

Panel no.	M = 1.61				M = 2.01			
	a(x,y)	b(x,y)	K'	K''	a(x,y)	b(x,y)	K'	K''
1	9.2697	-3.4345	1.0850	-0.4020	7.2906	-1.6314	1.0717	-0.2398
6	14.0488	.1310	.9867	.0092	11.6001	.3626	1.0556	.0330
11	17.9408	5.0386	.9094	.2554	15.6882	2.2197	1.0495	.1485
16	21.8285	10.2403	.8680	.4072	19.5233	3.6253	1.0372	.1926
21	24.9074	17.6248	.8157	.5772	22.4728	6.7974	.9918	.3000
2	11.5987	-12.6178	1.0175	-1.1069	7.9432	-5.7640	.8402	-.6097
7	17.3199	-11.3185	1.0023	-.6550	12.9534	-6.2884	.9206	-.4469
12	21.3807	-5.5358	.9505	-.2461	17.0295	-5.6898	.9434	-.3152
17	24.6134	2.1199	.8975	.0773	20.7659	-6.3633	.9549	-.2926
22	28.0989	11.9786	.8494	.3621	23.9667	-4.7914	.9479	-.1895
3	10.8329	-15.1296	.8630	-1.2053	7.0393	-6.7378	.6683	-.6397
8	17.3068	-17.9809	.9396	-.9762	11.5465	-8.3330	.7550	-.5449
13	22.2945	-16.4506	.9549	-.7046	15.3524	-11.0072	.7994	-.4847
18	26.6319	-13.1461	.9566	-.4722	19.0036	-10.6370	.8368	-.4684
23	29.9712	-7.9526	.9060	-.2404	22.3299	-11.0517	.8597	-.4255
4	8.6582	-13.0479	.6935	-1.0451	5.6727	-6.2845	.5385	-.5966
9	13.7726	-15.4485	.7643	-.8573	9.3070	-8.2784	.6159	-.5478
14	18.1381	-15.8221	.8059	-.7030	12.3551	-9.4272	.6583	-.5023
19	22.6245	-17.3120	.8543	-.6537	15.2406	-10.7046	.6938	-.4873
24	27.0432	-20.4038	.8969	-.6767	17.7668	-11.5272	.7139	-.4632
5	6.0293	-10.5993	.5396	-.9486	3.6773	-4.4132	.3888	-.4666
10	10.2350	-15.3493	.6410	-.9613	6.1707	-6.0818	.4580	-.4514
15	13.9321	-19.0233	.7052	-.9629	8.2881	-7.1979	.4987	-.4331
20	17.5026	-22.9774	.7596	-.9972	10.1703	-8.0126	.5262	-.4146
25	20.6906	-26.1415	.7994	-1.0100	11.4771	-7.2671	.5275	-.3340



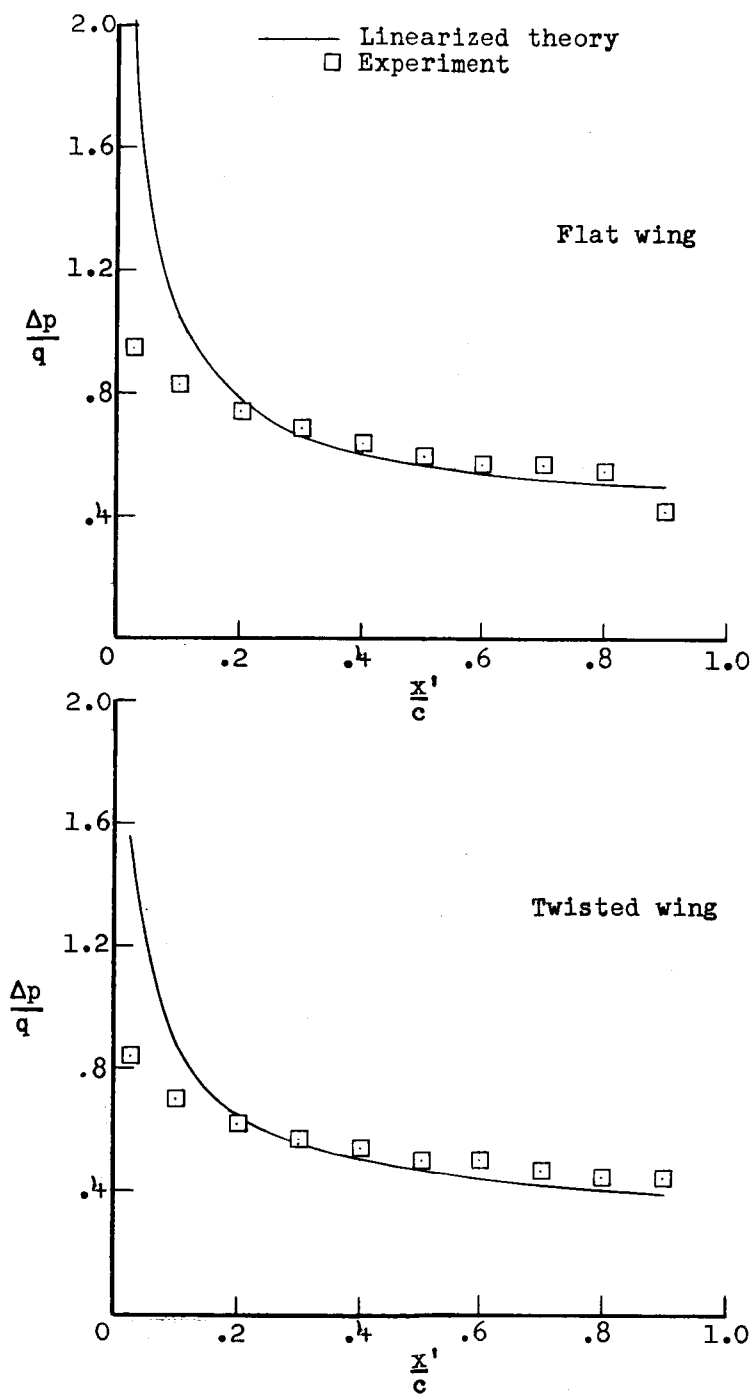
(a) 50 percent semispan; $\alpha_r = 6^\circ$.

Figure 1.- Chordwise distributions of lifting pressure on flat and twisted wings at $M = 1.61$.



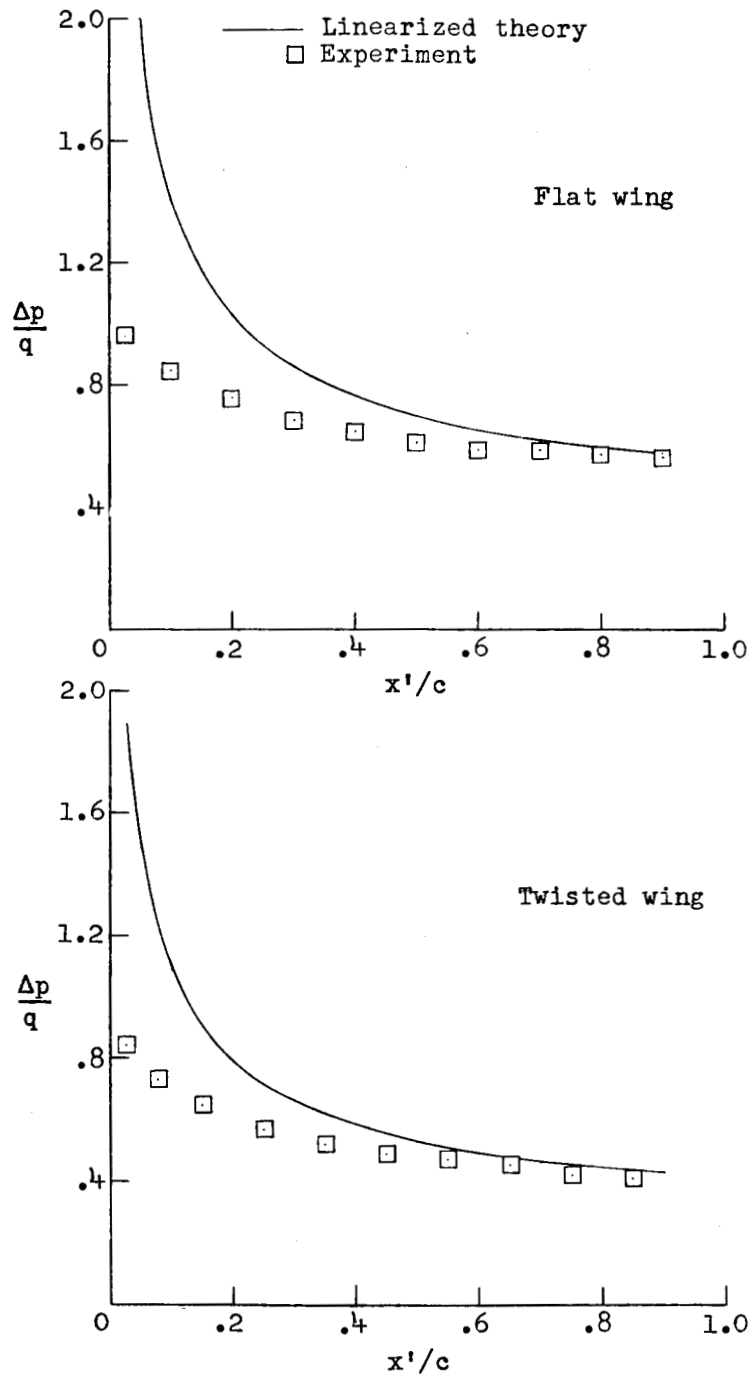
(b) 70 percent semispan; $\alpha_r = 6^\circ$.

Figure 1.- Continued.



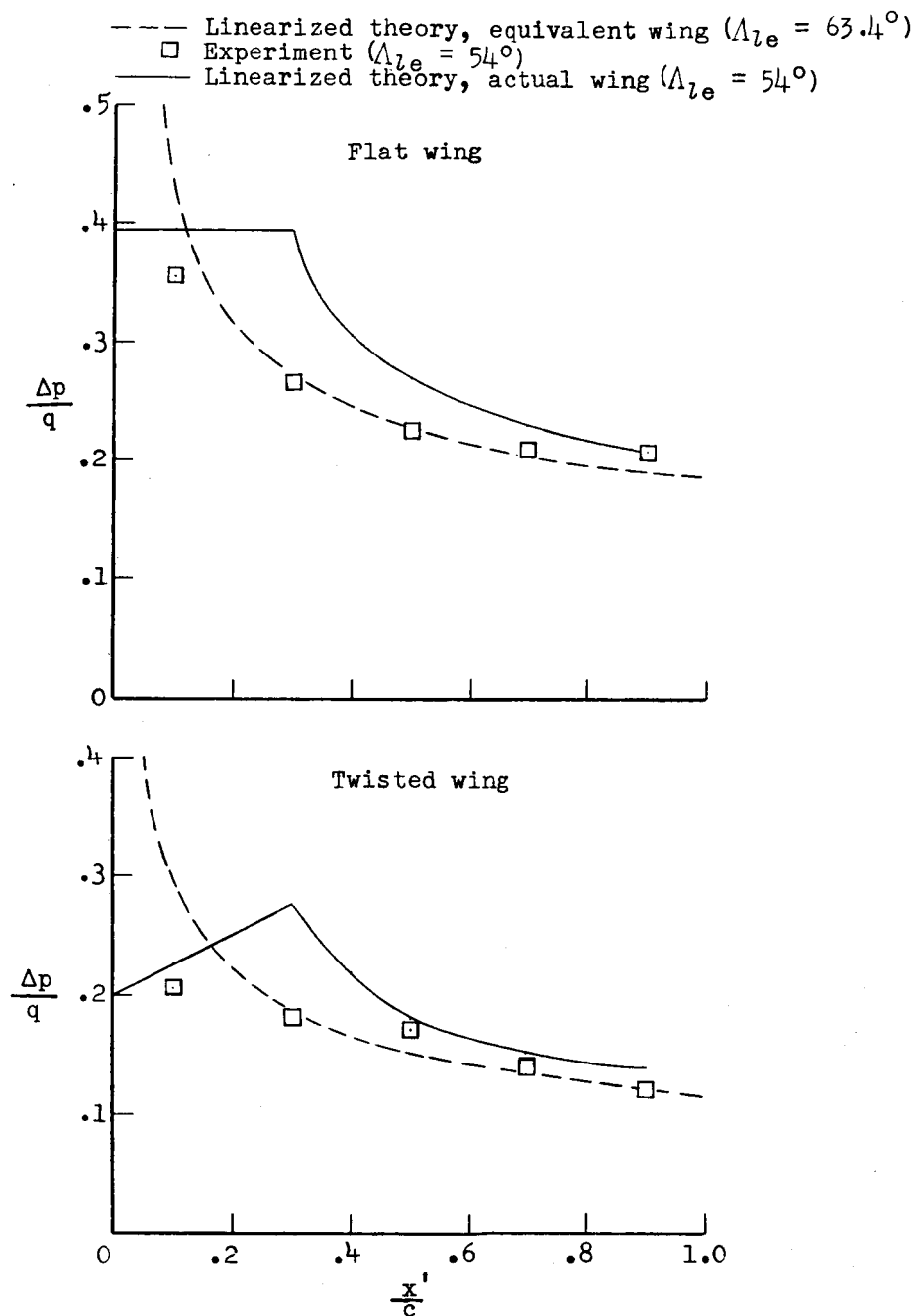
(c) 50 percent semispan; $\alpha_r = 12^\circ$.

Figure 1.- Continued.



(d) 70 percent semispan; $\alpha_r = 12^\circ$.

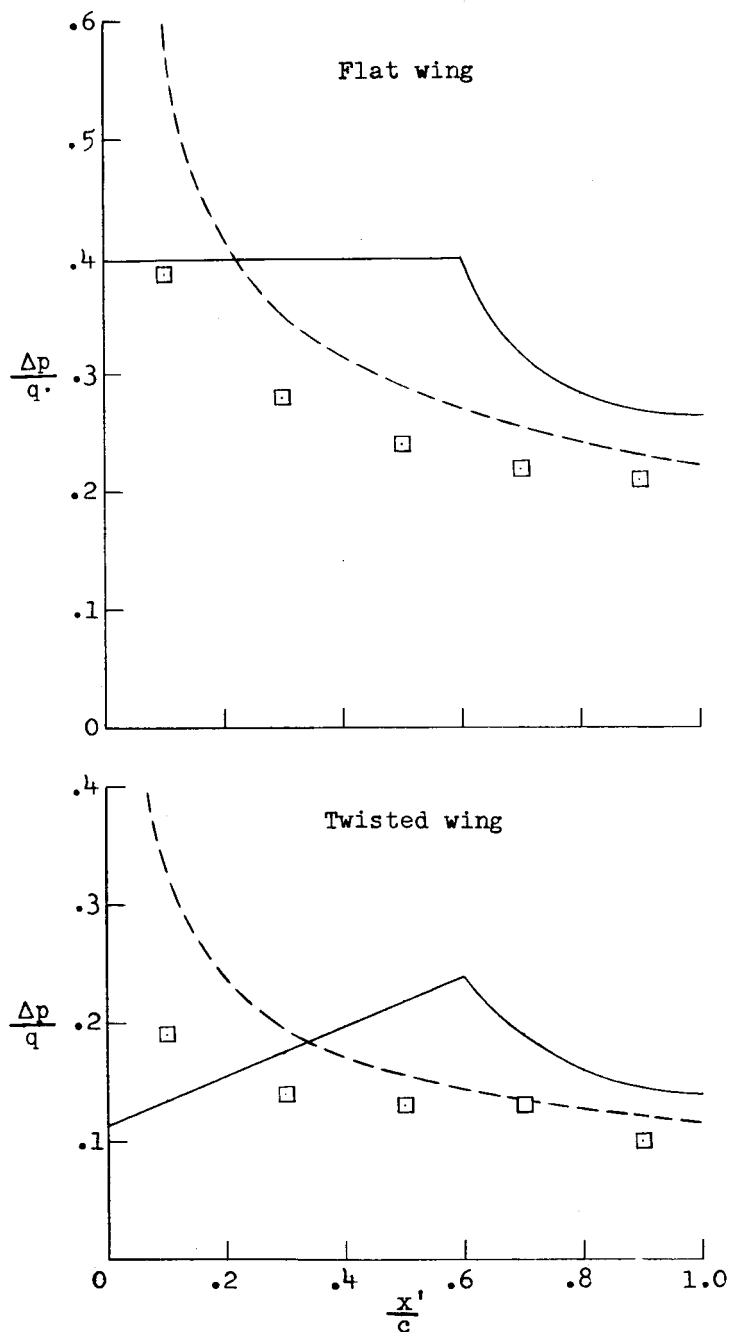
Figure 1.- Concluded.



(a) 50 percent semispan.

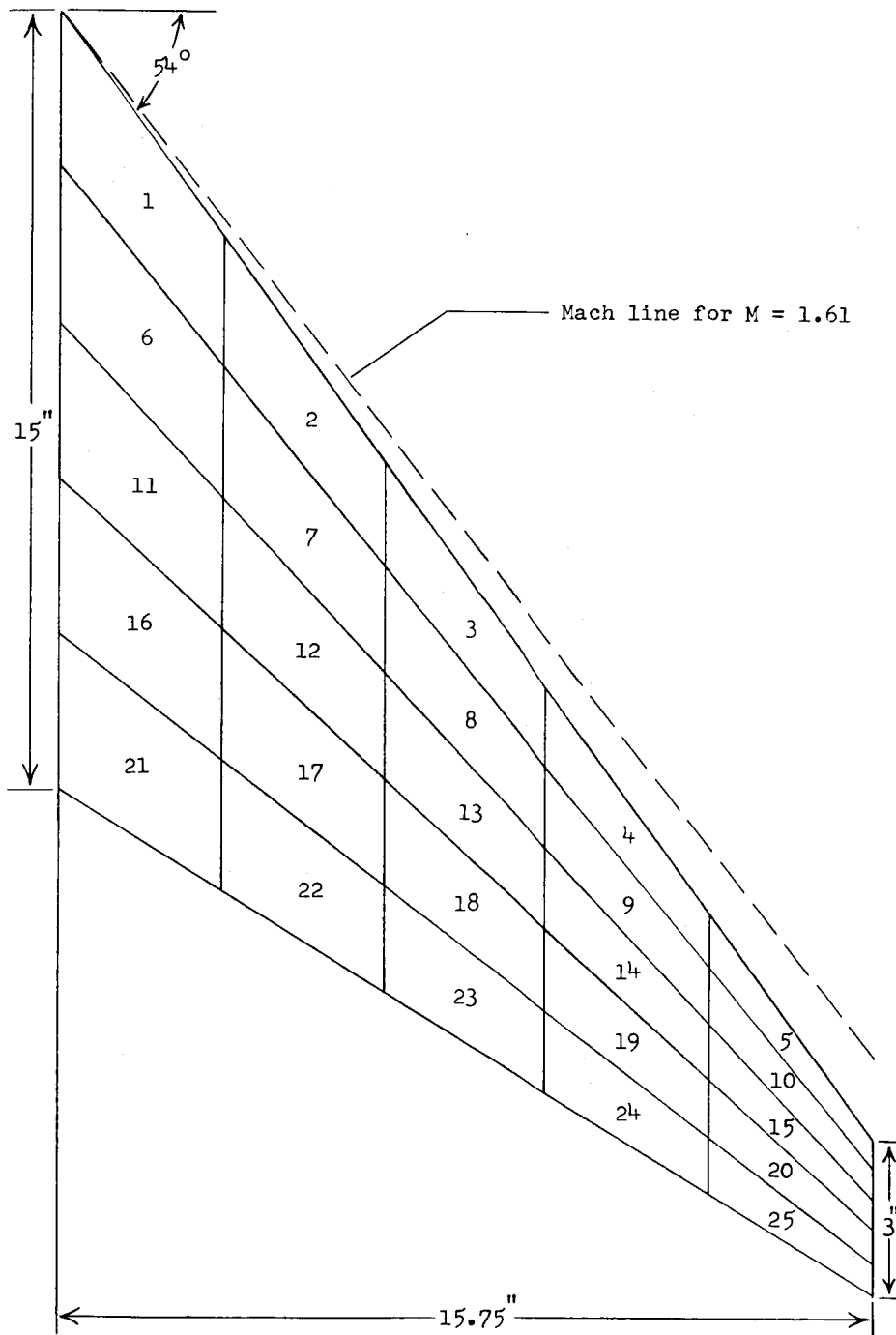
Figure 2.- Chordwise distribution of lifting pressure on flat and twisted wing at $M = 2.01$ and $\alpha_r = 6^\circ$.

--- Linearized theory, equivalent wing ($\Lambda_{te} = 63.4^\circ$)
 □ Experiment ($\Lambda_{te} = 54^\circ$)
 — Linearized theory, actual wing ($\Lambda_{te} = 54^\circ$)



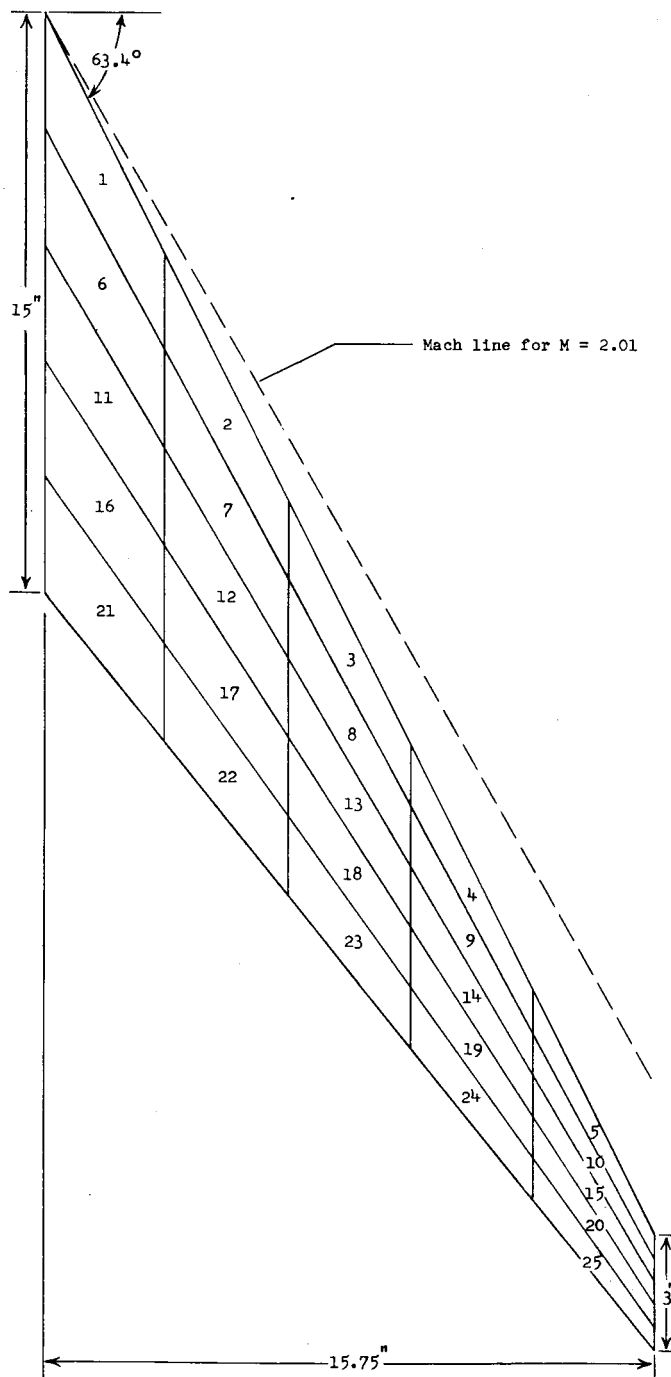
(b) 70 percent semispan.

Figure 2.- Concluded.



(a) Actual wing planform.

Figure 3.- Wing planform showing boundaries of influence panels.



(b) Equivalent wing planform for $M = 2.01$.

Figure 3.- Concluded.

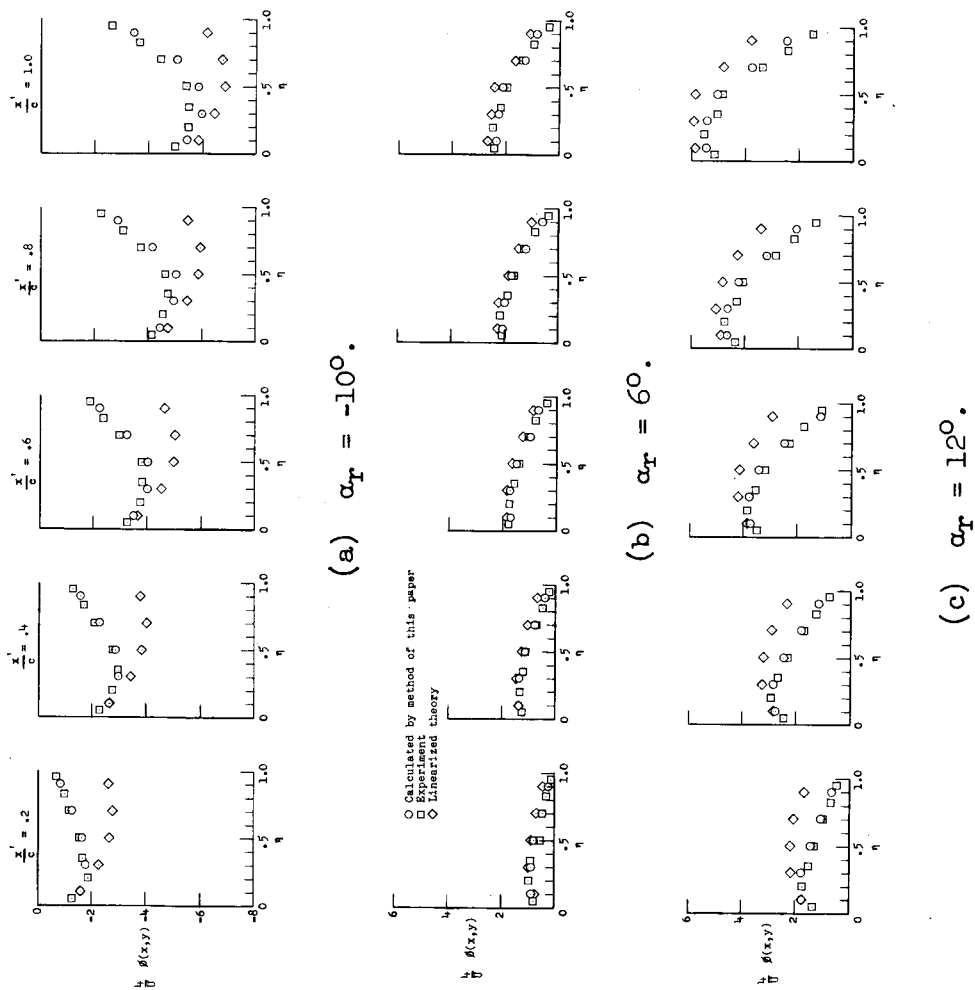


Figure 4.- Comparison of experimental and calculated spanwise distributions of $\frac{1}{b}\phi(x,y)$ at constant fractions of the chord for the twisted wing at $M = 1.61$, $\alpha_r = -10^\circ$, 6° , and 12° .

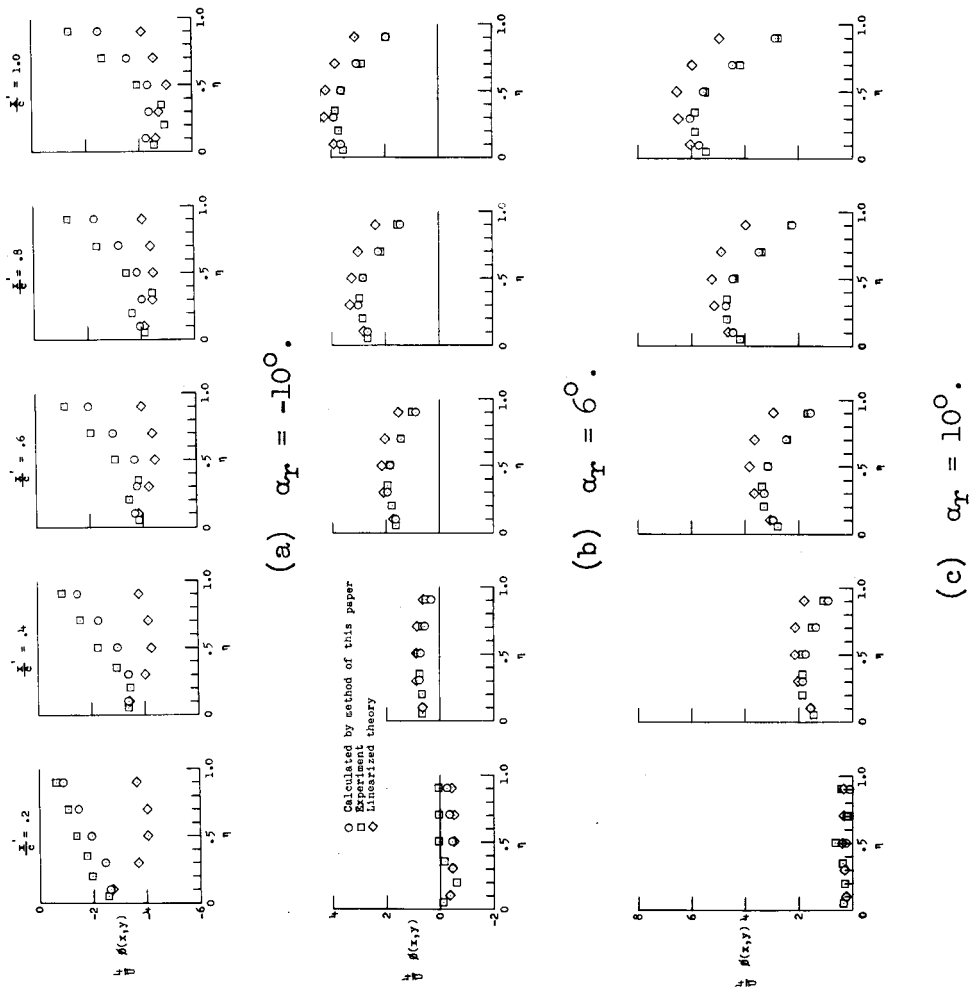


Figure 5.- Comparison of experimental and calculated spanwise distributions of $\frac{1}{4}\phi(x,y)$ at constant fractions of the chord for the cambered wing at $M = 1.61$, $\alpha_r = -10^\circ$, 6° , and 10° .

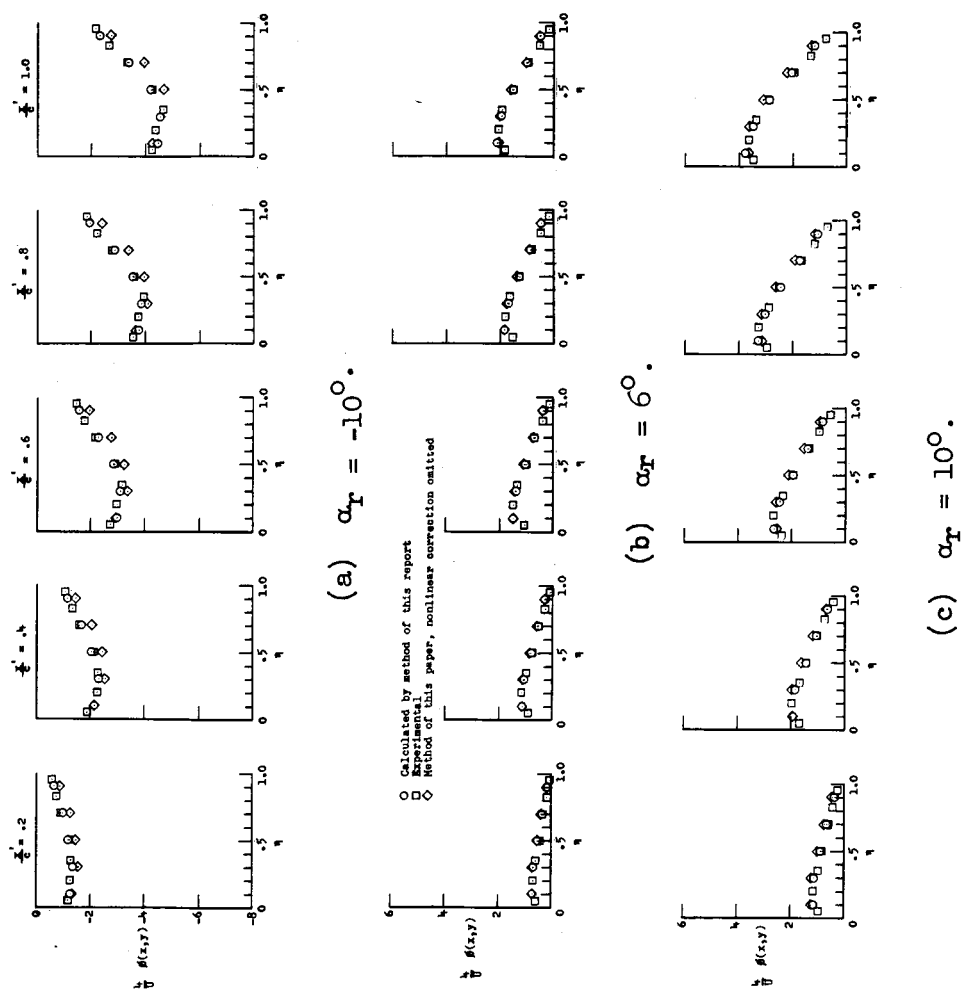


Figure 6.- Comparison of experimental and calculated spanwise distributions of $\frac{1}{2}\rho(x,y)$ at constant fractions of the chord for the twisted wing at $M = 2.01$, $\alpha_r = -10^\circ$, 6° , and 10° .

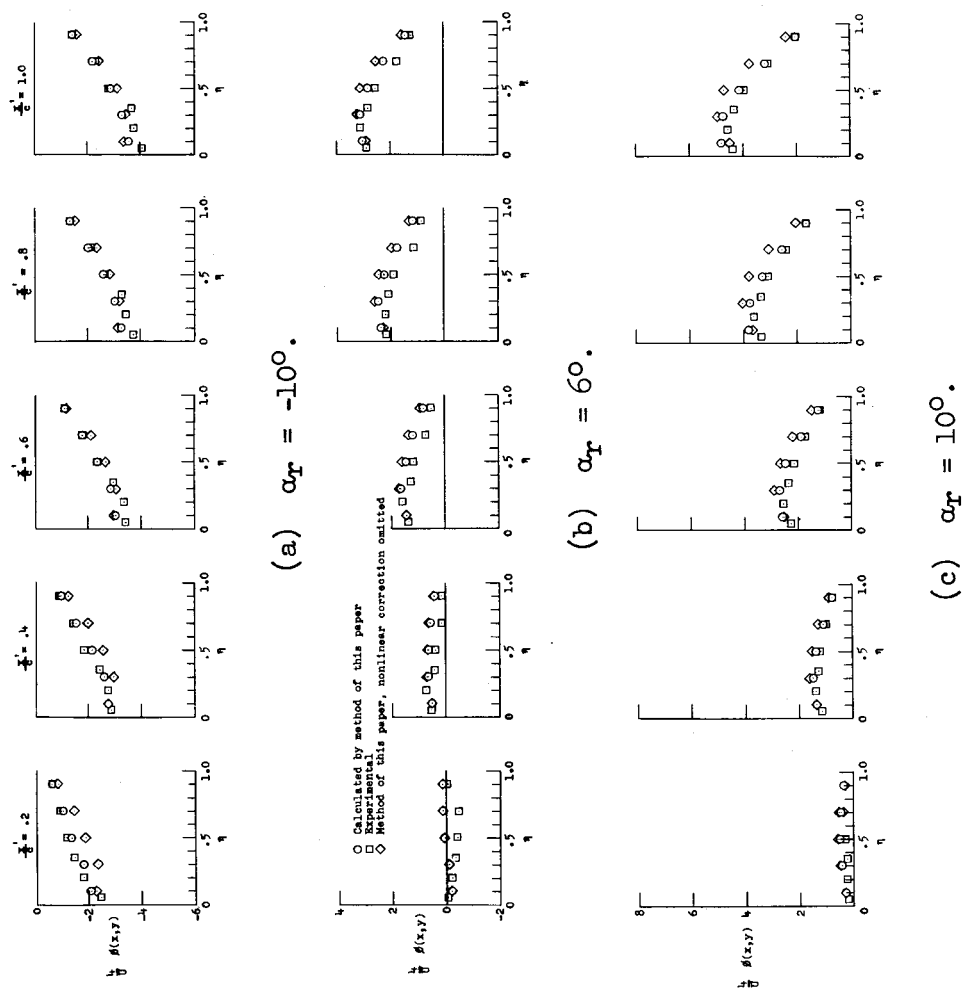


Figure 7.- Comparison of experimental and calculated spanwise distributions of $\frac{1}{2}\phi(x,y)$ at constant fractions of the chord for the cambered wing at $M = 2.01$, $\alpha_r = -10^\circ$, 60° , and 100° .